TEMPERATURE FIELD OF A COMPOSITE BODY UNDER MIXED THERMAL CONTACT CONDITIONS

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A heat-conduction equation is obtained for a piecewise-homogeneous body in the case when nonideal thermal contact conditions are satisfied on part of the connecting surface. The temperature field of a composite semiinfinite plate is investigated as an example.

Taking account of a different species of inclusion and defect on the interface of homogeneous elements is essential in investigating the temperature fields of piecewise-homogeneous structure elements. The influence of such a species of inclusion and interlayer defect can be simulated with sufficient accuracy by nonideal thermal contact conditions [1, 2]. An approach is proposed below to the solution of heat-conduction problems for composite bodies under inhomogeneous thermal contact conditions.

Let us consider a piecewise-homogeneous body consisting of two heterogeneous bodies joined along the surface z = d. Nonideal conditions are satisfied in the domain G of the connecting surface, while ideal thermal contact conditions are satisfied on the remaining surface. Let N(x, y) be the characteristic function of the domain G, such that

$$N(x, y) = \begin{cases} 1, & (x, y) \in G, \\ 0, & (x, y) \bar{\in} G. \end{cases}$$

We represent the temperature and thermophysical characteristics of the composite body in the form

$$p(z) = p_1(z) + [p_2(z) - p_1(z)]\, S_-(z-d), \tag{1}$$

where $p_i(z)$ (i = 1, 2) are characteristics of the i-th element of the system, and $S_-(z) = \begin{cases} 1, & z \geqslant 0, \\ 0, & z < 0. \end{cases}$

As a result of substituting the generalized conjugate problem [3] for the heat conduction equation in each of the composite parts of the system, we obtain the following equation:

$$\Delta t + \frac{\partial^2 t}{\partial z^2} - A(z)\frac{\partial t}{\partial \tau} = -\frac{W(x, y, z)}{\lambda(z)} + \delta_-(z-d)\left[\left.\frac{\partial t}{\partial z}\right|_{z=d+0} - \left.\frac{\partial t}{\partial z}\right|_{z=d-0}\right] + \delta'_-(z-d)[t|_{z=d+0} - t|_{z=d-0}]. \tag{2}$$

Here A(z) and λ(z) have the form (1): $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\delta_-^{(\kappa)}(z) = \frac{d^{\kappa+1} S_-(z)}{dz^{\kappa+1}}$, $A_k = 1/a_k$.

Having determined the temperature jump and its derivative on the connecting surface from the nonideal thermal contact conditions [2] by using the characteristic function of the domain G, and having substituted their values into the right side of (2), we obtain a partially degenerate differential equation for the heat conduction of the composite body

$$\Delta t + \frac{\partial^2 t}{\partial z^2} - A(z)\frac{\partial t}{\partial \tau} = -\frac{W(x, y, z)}{\lambda(z)} + \delta_-(z-d)\left[\left.\frac{\partial t}{\partial z}\right|_{z=d-0}(1 - K_\lambda^{(1)}) - \right. \tag{3}$$

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$$-\frac{N(x,\ y)}{2\lambda_1}\left(\Lambda_0\Delta-C_0\frac{\partial}{\partial\tau}\right)(t|_{z=d+0}+t|_{z=d-0})\Bigg]+\delta_-'(z-d)N(x,\ y)\frac{R_0}{6}\Bigg[\Bigg(\Lambda_0\Delta- \tag{3}$$

$$-C_0\frac{\partial}{\partial\tau}\Bigg)(2t|_{z=d+0}+t|_{z=d-0})+6\lambda_2\frac{\partial t}{\partial z}\Bigg|_{z=d+0}\Bigg],$$

where $\Lambda_0=2\lambda_0 h;\ C_0=2c_v^{(0)}h;\ K_\lambda^{(i)}=\lambda_2/\lambda_i;\ R_0=2h/\lambda_0$.

The heat-conduction equation for a piecewise-homogeneous plate can also be obtained in an analogous manner. Let a composite plate consist of two joined heterogeneous plates of thickness 2δ. Nonideal thermal contact conditions [2] are satisfied on the sections $(a^{(1)}{}_i,\ a^{(2)}{}_i)$ $(i=\overline{1,\ N})$ of the connecting surface $x=\ell$, while heat transfer with the external medium of temperature t_c is accomplished from the side surfaces of the plate. Using the nonideal thermal contact condition [1], we obtain a partially degenerate differential equation with piecewise-constant coefficients to determine the integrated characteristics of the temperature:

$$\Delta T-\varkappa^2(x)(T-t_c)-A(x)\frac{\partial T}{\partial\tau}=-\frac{W(x,\ y)}{\lambda(x)}$$

$$+\delta_-(x-l)\left\{\frac{\partial T}{\partial x}\Bigg|_{x=l+0}(1-K_\lambda^{(1)})-\right.$$

$$-\frac{N(x)}{2\Lambda_1}\left[\left(\Lambda_0\frac{\partial^2}{\partial y^2}-2A_0-C_0\frac{\partial}{\partial\tau}\right)(T|_{x=l+0}+T|_{x=l-0})+4A_0\Theta_c\right]\Bigg\}+ \tag{4}$$

$$+\delta_-'(x-l)N(x)\frac{R_h}{6}\left[6\Lambda_2\frac{\partial T}{\partial x}\Bigg|_{x=l+0}+\right.$$

$$\left.+\left(\Lambda_0\frac{\partial^2}{\partial y^2}-C_0^*\frac{\partial}{\partial\tau}-2A_0\right)(2T|_{x=l+0}+T|_{x=l-0})+2A_0(3\Theta_c+\Theta_c^*)\right].$$

Here, $\Lambda_i=2\delta\lambda_i,\ N(x)=\sum_{\kappa=1}^{N}[S_+(x-a_\kappa^{(1)})-S_-(x-a_\kappa^{(2)})],\ A_0=2\alpha_z^{(0)}h,\ R_h=h/\lambda_0\delta,\ \varkappa_i^{(2)}=\alpha_z^{(i)}/\lambda_i\delta.$

As an illustration we determine the stationary temperature field in a semiinfinite plate connected to a strip of width ℓ heated along the endface by an external medium of temperature t_c. Heat transfer with the external medium of zero temperature is accomplished from the side surfaces of the plate, and all the thermophysical characteristics of the half-plane and the strips are distinct. Nonideal thermal contact conditions are satisfied on the section $(-a,\ a)$ of the connecting surface $x=\ell$. Neglecting terms of lower order of smallness with respect to h in (4), we arrive at an equation taking account of just the thermal resistance on part of the contact surface

$$\Delta T-[\varkappa_1^2+(\varkappa_2^2-\varkappa_1^2)S_-(x-l)]T=$$

$$=R_0\lambda_2\frac{\partial T}{\partial x}\Bigg|_{x=l+0}[S_+(y+a)-S_-(y-a)]\delta_-'(x-l)+(1-K_\lambda^{(1)})\frac{\partial T}{\partial x}\Bigg|_{x=l+0}\delta_-(x-l). \tag{5}$$

The boundary conditions have the form

$$T|_{x\to\infty}=0;\quad T|_{x=0}=t_c. \tag{6}$$

Let us represent the function $\left.\frac{\partial T}{\partial x}\right|_{x=l+0}$ as a series in the interval $(-a,\ a)$

$$\frac{\partial T}{\partial x}\Bigg|_{x=l+0}=\sum_{n=0}^{\infty}\beta_n\cos p_n y, \tag{7}$$

where

$$\beta_n=\frac{1}{a}\int_{-a}^{a}\frac{\partial T}{\partial x}\Bigg|_{x=l+0}\cos p_n y\,dy. \tag{8}$$

Since we have $\left.\frac{\partial T}{\partial x}\right|_{x=l+0} \xrightarrow[y\to\pm a\mp 0]{} 0$ from the contact condition and the continuity of the temperature in the coordinate y, then $p_n = \left(n+\frac{1}{2}\right)\pi a$.

Applying the Fourier transform in y to (5) and (6) with (7) taken into account, we obtain

$$\frac{d^2\overline{T}}{dx^2} - [\gamma_1^2 + (\gamma_2^2 - \gamma_1^2) S_-(x-l)]\,\overline{T} = (1 - K_\lambda^{(1)})\left.\frac{\partial T}{\partial x}\right|_{x=l+0} \delta_-(x-l) + \tag{9}$$

$$+ 2R_0\lambda_2\delta_-'(x-l)\cos\eta a \sum_{j=0}^{\infty} \beta_j p_j \frac{(-1)^j}{p_j^2 - \eta^2},$$

$$\overline{T}|_{x\to\infty} = 0,\ \overline{T}|_{x=0} = 2\pi t_c \delta(\eta). \tag{10}$$

The general solution of the homogeneous equation corresponding to (9) has the form [4]

$$\overline{T}_0 = \sum_{i=1}^{2} C_i \Big\{\exp(k_i\gamma_1(x-l)) + [\exp(k_i\gamma_2(x-l)) - \exp(k_i\gamma_1(x-l))]\,S_-(x-l) -$$

$$- k_i(\gamma_2 - \gamma_1)\frac{\operatorname{sh}\gamma_2(x-l)}{\gamma_2} S_-(x-l)\Big\},\quad k_i = \begin{cases} 1, & i=1, \\ -1, & i=2. \end{cases}$$

We select the following as a particular solution of (9):

$$\overline{T}_p = [S_-(x-l) - 1]\left[u_1 \operatorname{ch}\gamma_1(x-l) + u_2\frac{1}{\gamma_1}\operatorname{sh}\gamma_1(x-l)\right],$$

$$u_1 = 2R_0\lambda_2\cos\eta a\sum_{n=0}^{\infty}\beta_n p_n\frac{(-1)^n}{p_n^2 - \eta^2},$$

$$u_2 = (1 - K_\lambda^{(1)})\left.\frac{dT}{dx}\right|_{x=l+0}.$$

The general solution of (9) is

$$\overline{T} = \overline{T}_0 + \overline{T}_p.$$

Satisfying the boundry conditions (10) and applying the inverse Fourier transform, we obtain a system of linear equations from (8) to determine the unknown coefficients:

$$A_{j\kappa}\beta_\kappa^* = B_j,\ \kappa,\ j = \overline{0,\ \infty}, \tag{11}$$

where

$$A_{j\kappa} = \frac{4hK_\lambda^{(0)}}{\pi a} p_j p_\kappa (-1)^j \int_0^\infty \frac{\gamma_1\gamma_2\cos^2 a\eta\, d\eta}{(\gamma_1 + K_\lambda^{(1)}\gamma_2 \operatorname{th}\gamma_1 l)(p_\kappa^2 - \eta^2)(p_j^2 - \eta^2)} + \frac{1}{2}(-1)^j\delta_{\kappa j},$$

$$B_j = -\frac{\varkappa_2^2\varkappa_1^2}{ap_j(\varkappa_1^2 \operatorname{ch}\varkappa_1^2 l + \varkappa_2^2 K_\lambda^{(1)} \operatorname{sh}\varkappa_1^2 l)},$$

$$\beta_\kappa^* = \beta_i/t_c,\quad \gamma_\kappa^2 = \varkappa_\kappa^2 + \eta^2.$$

The quasiregularity of the infinite system of linear equations (11) was investigated numerically for different parameters of the problem.

The temperature field of the piecewise-homogeneous plate under consideration has the form

$$T = t_c T^*,$$

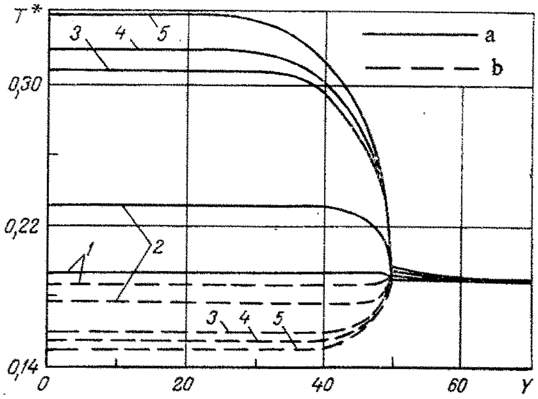

Fig. 1

where

$$T^* = [S_-(x-l)-1]\Bigg[-\frac{\varkappa_1^2 \operatorname{ch}\varkappa_1^2(x-l) - K_\lambda^{(1)}\varkappa_2^2 \operatorname{sh}\varkappa_1^2(x-l)}{\varkappa_1^2 \operatorname{ch}\varkappa_1^2 l + \varkappa_2^2 K_\lambda^{(1)} \operatorname{sh}\varkappa_1^2 l} +$$

$$+\frac{4HK_\lambda^{(0)}K_\lambda^{(1)}}{\pi}\sum_{n=0}^{\infty}\beta_n^* p_n(-1)^n \int_0^\infty \frac{\gamma_2 \cos a\eta \cos y\eta}{(\gamma_1 + K_\lambda^{(1)}\gamma_2 \operatorname{th}\gamma_1 l)(p_n^2-\eta^2)}\frac{\operatorname{sh}\gamma_1 x}{\operatorname{ch}\gamma_1 x}d\eta\Bigg] \tag{12}$$

$$+S_-(x-l)\Bigg[\frac{\varkappa_1^2 \exp(-\varkappa_2^2(x-l))}{\varkappa_1^2 \operatorname{ch}\varkappa_1^2 l + \varkappa_2^2 K_\lambda^{(1)} \operatorname{sh}\varkappa_1^2 l} - \frac{4HK_\lambda^{(0)}}{\pi}\sum_{n=0}^{\infty}\beta_n^* p_n(-1)^n \int_0^\infty \frac{\gamma_1 \cos a\eta \cos y\eta \exp(-\gamma_2(x-l))}{(\gamma_1 + \gamma_2 K_\lambda^{(1)} \operatorname{th}\gamma_1 l)(p_n^2-\eta^2)}d\eta\Bigg].$$

Substituting $b_k = K^{(0)}{}_\lambda \beta^*{}_k$ in the relationships (11) and (12), and passing to the limit as $K^{(0)}{}_\lambda \to \infty$, we arrive at a result corresponding to the case of thermal insulation on part of the connecting surface. In this case we have a system of algebraic equations

$$A_{j\kappa}^* b_\kappa = B_j,$$

where

$$A_{j\kappa}^* = \frac{4h}{\pi a} p_j p_\kappa (-1)^j \int_0^\infty \frac{\gamma_1\gamma_2 \cos^2 a\eta d\eta}{(\gamma_1 + K_\lambda^{(1)}\gamma_2 \operatorname{th}\gamma_1 l)(p_\kappa^2-\eta^2)(p_j^2-\eta^2)}.$$

The dimensionless temperature field is computed by means of (12) for different values of the parameter $K^{(0)}{}_\lambda$. The dependence of the value T* on the interface $X = x/\delta = L$ on the dimensionless coordinate $Y = y/\delta$ is represented in Fig. 1 for $A = a/\delta = 50$, $L = \ell/\delta = 50$, $K^{(1)}{}_\lambda = 2$, $Bi_1 = 0.01$, $Bi_2 = 0.04$, $H = h/\delta = 1$. Curves 1-4 are constructed, respectively, for the values $K^{(0)}{}_\lambda = 1, 5, 50, 100$; curve 5 corresponds to the case of a heat insulated section. It is seen from Fig. 1 that the quantity $K^{(0)}{}_\lambda$ does not influence the temperature of the contact surface outside the inclusion substantially. As $K^{(0)}{}_\lambda$ increases, the magnitude of the temperature jump grows, where the maximum value of the jump is reached in the thermal insulation case.

NOTATION

x, y, z, Cartesian coordinates; t, temperature; τ, time; h, thickness of the inclusion; $S_\pm(x)$, asymmetric unit Heaviside functions; W(x, y, z), heat source distribution density; λ_0, λ_i, heat-conduction coefficients of the inclusion and the i-th element of the system; $c^{(0)}{}_v$, bulk specific heat of the inclusion; 2δ, plate thickness; T, integrated temperature characteristic; R_0, thermal resistance of the inclusion; $\alpha^{(i)}{}_z$, heat transfer coefficient from the surfaces $z = \pm\delta$ of the i-th element of a composite plate; X, Y, dimensionless Cartesian coordinates; $Bi_i = \alpha^{(i)}{}_z\delta/\lambda_i$, Biot criterion for the i-th element of a plate; a_i, thermal diffusivity coefficient for the i-th element of the system.

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POTENTIALS OF THE PROBLEM OF STEADY-STATE OSCILLATIONS OF THE GENERALIZED ASYMMETRICAL THERMOELASTICITY OF A COSSERAT MEDIUM

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The potentials of a simple layer and a double layer are determined, along with the volume potential of the problem of steady-state oscillations of the generalized thermoelasticity of a Cosserat medium; these potentials lead to integral equations of the second kind for the problem.

In the investigation of laser-induced thermal strains of optical materials it is necessary to treat more complex models than the classical versions in some cases. First of all, the finiteness of the heat propagation velocity must be considered in the study of heat-release processes associated with the pulsed application of optical radiation, i.e., it is necessary to go from classical to generalized thermomechanics [1]. Second, when the application of such materials as polycrystalline aggregates or an optical ceramic is considered, it is required to include not only the regular microstresses, but also couple stresses [2], necessitating the introduction of the Cosserat continuum model. The generalized thermoelasticity equations for a Cosserat medium have been derived previously [3]. An important special case is the problem of steady-state harmonic oscillations of a homogeneous isotropic polar-symmetrical medium. The system of equations in the complex amplitudes of the kinematic variables for an oscillation with frequency σ has the form

$$\begin{gathered}(\mu+\alpha)\nabla^2\mathbf{u}+(\mu+\lambda-\alpha)\nabla\nabla\cdot\mathbf{u}+2\alpha\nabla\times\boldsymbol{\omega}-\nu\Theta_0\nabla\vartheta+\rho\sigma^2\mathbf{u}+\mathbf{X}=0,\\ (\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\nabla\nabla\cdot\boldsymbol{\omega}+(I\sigma^2-4\alpha)\boldsymbol{\omega}+2\alpha\nabla\times\mathbf{u}+\mathbf{Y}=0,\\ \frac{\Theta_0 k}{1+i\sigma\tau_0}\nabla^2\vartheta-i\sigma m\Theta_0^2\vartheta-i\sigma\Theta_0\nu\nabla\cdot\mathbf{u}+w=0.\end{gathered} \tag{1}$$

One of the methods of analyzing and solving the boundary-value problems of thermoelasticity, particularly for regions bounded by noncanonical surfaces, is to reduce them to integral equations [4-6], specifically by means of potentials. It is first of all necessary in this connection to formulate the fundamental solutions [6] of the system (1).

We consider the problem of the action of a point force vector with amplitude value $\mathbf{a}_1$ applied at the origin for Y = 0, w = 0. Invoking the regular solution of the homogeneous system (1) and the Fourier integral transform for the formulation of a particular solution of the inhomogeneous system, we obtain a solution of the system (1) subject to the Sommerfeld radiation condition:

$$\mathbf{u}=\mathbf{U}^{(1)}\cdot\mathbf{a}_1,\quad \boldsymbol{\omega}=\boldsymbol{\Omega}^{(1)}\cdot\mathbf{a}_1,\quad \vartheta=\boldsymbol{\Theta}^{(1)}\cdot\mathbf{a}_1. \tag{2}$$

Here the tensors $U^{(1)}$, $\Omega^{(1)}$ and the vector $\Theta^{(1)}$ have the form

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